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Thermal instabilities in a horizontal cylindrical duct: a physical approach

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Abstract

Flow of water in a uniformly heated horizontal cylindrical duct induces a mixed convection phenomenon. For adapted coupling of the Reynolds and Rayleigh number values, an instability is reached for which fluctuations of great amplitude in the wall temperature occur sporadically. The aim of this paper is to analyze this thermal instability and to propose a physical mechanism which governs this kind of instability. Indeed, we show that this instability is the result of an exchange of the heat transfer mode from convective to diffusive in the thermal boundary layer, in the bottom of the cross section.

Résumé

L'écoulement de l'eau dans un conduit cylindrique horizontal et uniformément chauffé est le siège d'une convection mixte. Pour des valeurs adaptées des nombres de Reynolds et de Rayleigh, une instabilité se manifeste par des fluctuations sporadiques et de grande amplitude de la température de paroi. L'objet de cet article est d'analyser ce phénomène et de proposer un mécanisme physique qui gouverne ce type d'instabilité. En effet, nous montrons que celle-ci est la conséquence d'un échange du mode de transfert de chaleur de convectif à diffusif dans la couche limite thermique, en bas d'une section droite, puis retour à l'état d'origine.

1. Introduction

Mixed convection flow in a horizontal cylindrical heated duct is a well known phenomenon [1–3]. The buoyancy effect induces two contra-rotating convective rolls in a straight section; there follows the creation of a temperature difference between the top and the bottom of a straight cross section.

The laminar regime includes two kinds of flow. The first one, obtained for low Reynolds (*Re*) and Rayleigh (*Ra*) numbers, is the stable state and the second one, obtained when the Reynolds and/or Rayleigh numbers are increased, is the instabilities regime. An instability diagram [4], established using the wall temperature measurement, shows the location of these two states called "stable regime" and "instabilities regime". The stable regime has largely been described elsewhere [5–9]. The aim of this paper is to propose the physical mechanism governing this kind of instability.

2. Description of the "instability regime"

Our experiments concern the flow of distilled water in a uniformly heated horizontal cylindrical duct. The experimental set up has been described elsewhere [4]. For an adapted coupling of the Reynolds and Rayleigh numbers [4], the instability regime appears. It manifests itself through large amplitude fluctuations [10–12]. Indeed, the measurement of the wall temperature, on the top of cross section, "Th", shows successive fluctuations separated by various durations of stable laminar phases. The fluctuations are similar and are characterized by

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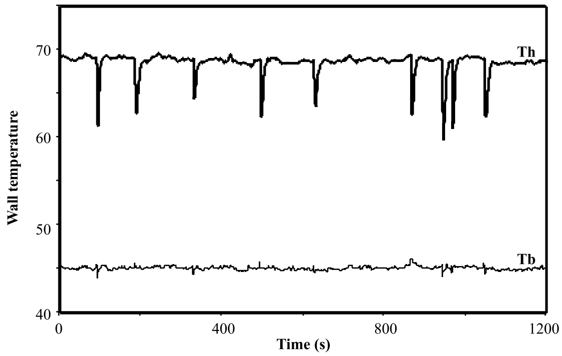

Fig. 1. Evolution of the wall temperature on the top (Th) and the bottom (Tb) of a cross section at an axial coordinate $z = 80$ cm ($Re = 1800$, $Ra = 250\,000$).

two time constants; the first, of an order of 1 s, corresponds to the temperature drop (cooling phase) and the second, of an order of 10 s, corresponds to the return to the laminar phase (heating phase). To illustrate this, Fig. 1 shows the evolution of the wall temperature on the top "Th" and on the bottom "Tb" of the cross section.

The occurrence of these fluctuations is random and corresponds to a modification of the thermal-hydrodynamical structure of the fluid. In order to emphasize this fact and to find the physical mechanism which governs this instability, measurements of temperature were made in the fluid. Thus a thermocouple was inserted in the fluid, at the outlet of the heated zone, within a straight section S at an axial coordinate $zs = 120$ cm ($z = 0$ corresponds to the beginning of the heated zone). The thermocouple position is marked in the plane S by r and θ, the radial and the azimuthal coordinates respectively.

To illustrate this, Figs. 2 and 3 show two typical examples of the evolution of the wall temperature Th (at an axial coordinate $z = 80$ cm) and Tclb at $zs = 120$ cm where Tclb is the fluid temperature at a point of the thermal boundary layer at the bottom of the straight

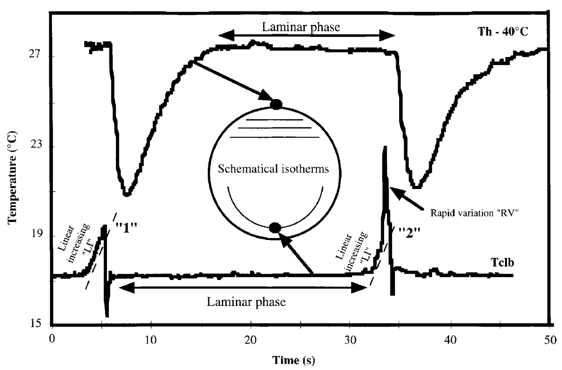

Fig. 2. Evolution of the wall temperature Th and the fluid temperature Tclb ($Re = 1720$, $Ra = 303\,400$).

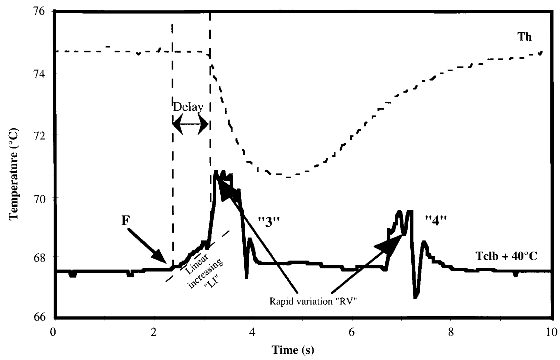

Fig. 3. Evolution of the wall temperature Th and the fluid temperature Tclb ($Re = 1586$, $Ra = 34\,600$).

section on the vertical diameter of section S. The signal of Tclb was shifted back in order to take into account the difference $[zs - z80]$ corresponding to the sections concerned.

We notice three kinds of fluctuations in the fluid temperature:

α – a linear increase, see fluctuation "1" of Fig. 2,

β – a linear increase followed by a rapid variation "RV", see fluctuation "2" of Fig. 2 and fluctuation "3" of Fig. 3,

γ – a rapid variation "RV" without the linear increase, see fluctuation "4" of Fig. 3.

The fluctuation of the wall temperature Th is related to the fluid fluctuation only in cases 1 and 2. In case 3, the top wall temperature does not exhibit any modification and remains stable. These facts demonstrate that the large amplitude fluctuations in the wall temperature exist only in the case where there is a linear increase of Tclb, i.e. fluctuation of Th is a response to the linear increase of Tclb. The amplitude of Th fluctuation is correlated to the duration of the linear increase of Tclb: the shorter the duration, the smaller the amplitude.

In Fig. 3, we marked F the time where a modification in the stable state occurs (for Tclb, it is the time corresponding to the beginning of linear increase); this time F could be located for the whole cross section, i.e. for all points (r, θ) of the cross section S. Then we can measure the time difference between this instant (corresponding to F) and the occurrence time of the fluctuation of Th at the axial coordinate $z = 80$ cm (the latter will be considered as a reference time). The set of the points $F(r, \theta)$ characterizes the occurrence time of the modification in the stable state in the whole cross section. So, using experimental data, Fig. 4 shows schematically the isochronous curves (curves with the same time difference related to the reference time); the standard deviation for an isochronous curve is small (less than 0.1 s).

The isochronous curves have the same shape as the isotherms in the fluid. They show that the instability phenomenon begins at first in the middle of the thermal

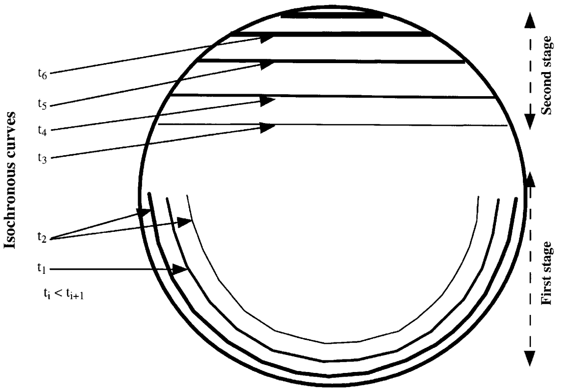

Fig. 4. Isochronous curves in a straight cross section.

boundary layer in the lower part of the cross section (curve t_1) then it propagates radially, on one hand towards the wall and on the other hand towards the cold core of the fluid (curves t_2). The propagation of the perturbation towards the upper part of the cross section, i.e. the thermal stratified zone, occurs in a second stage towards the curves t_3, t_4, t_5 and t_6 successively; the wall at the top of the cross section is the last to be affected by the perturbation and it is only at this time that the large amplitude fluctuation of Th occurs.

3. Analysis of the phenomenon

By analyzing the experimental results, we can propose a physical mechanism governing this kind of instability.

3.1. Phenomenological approach

We have already seen that the occurrence of a fluctuation in the wall temperature is a consequence of the linear increase in temperature of the thermal boundary layer in the lower part of the cross section. In order to simplify, we will consider only the lower part of the cross section for which the velocities and temperature fields are practically axisymmetrical; the cylindrical coordinates θ, r and z, the azimuthal, radial and axial coordinates respectively, are used.

3.1.1. Stable regime

In the stable regime and contrary to a Poiseuille flow in a heated duct where the heat transfer in a cross section is purely conductive (forced convection), in mixed convection phenomenon the heat supplied to the wall in the lower part of the cross section is transferred by the transverse velocities (secondary flow) to the upper part of the cross section; heat is advected by the transverse component of the velocity $v_\theta(\theta, r, z)$. In this case, we say that the heat transfer is convective (free convection) in the lower part of a cross section; if we consider a slice of fluid, of dz thickness, in the lower part of the cross section there is no accumulation of internal energy. Indeed, experimental results show that $\partial T_{\mathrm{clb}}/\partial t = 0$. Elsewhere [8,9], the numerical and experimental results show that the temperature of the boundary layer in the lower part of the cross section is constant when z increases, i.e. $\partial T_{\mathrm{clb}}/\partial z = 0$. This means that, in this zone, there is no advection of heat in the axial direction since $v_z(\theta, r, z)(\partial T_{\mathrm{clb}}/\partial z) = 0$. So, in the stable regime, in the lower part of the cross section, heat transfer is convective (free convection in the cross section) from the lower part to the upper part. However for the upper part heat transfer is diffusive (forced convection in the axial direction).

3.1.2. Instability regime

When the instability occurs, experimental results show that in the thermal boundary layer (lower part of the cross section) $\partial T_{\mathrm{clb}}/\partial t$ is different from zero; in this case there is an accumulation of internal energy in this zone, that is: $\rho C_{\mathrm{p}}(\partial T_{\mathrm{clb}}/\partial t) \neq 0$. Thus, we find a situation which is analogous to the case of a heated Poiseuille flow (forced convection, in the thermal transient phase, where versus time, we have increasing of internal energy of fluid and in a parallel fashion, creation of an axial temperature gradient, that is $\partial T_{\mathrm{clb}}/\partial z \neq 0$ [13]). We can deduce that for a slice of fluid, of dz thickness, in the lower part of the cross section, heat advection in the axial direction z is not zero since $v_z(\theta, r, z)(\partial T_{\mathrm{clb}}/\partial z) \neq 0$. This increase in internal energy, in the lower part, may take place only if free convection becomes unable to totally advect heat towards the upper part of the cross section (contrary to the stable regime). So, heat supplied by the wall, in the lower part, is transferred radially by thermal diffusion in the thermal boundary layer towards the center of the cross section. This thermal diffusion which was neglected in the stable regime in relation to free convection, becomes an effective mode of heat transfer. Schematically, for an increase in internal energy to occur and for diffusion to be preponderant it suffices that transverse velocities no longer evolve versus z (or that their evolution versus z becomes less significant than in the stable regime, i.e. $(\partial v_\theta(\theta, r, z)/\partial z)_{\mathrm{stable}} > (\partial v_\theta(\theta, r, z)/\partial z)_{\mathrm{unstable}}$), or that in the extreme limit $(\partial v_\theta(\theta, r, z)/\partial z)_{\mathrm{unstable}} = 0$; this behavior is limited to the slice of fluid (of dz thickness) concerned by the instability.

Thus, each transition (point F of Fig. 3), corresponding to the beginning of the linear increase in the lower part, is the sign of a sudden change in heat transfer mode in the boundary layer from a convective mode to a diffusive one. The increase in internal energy and the creation of an axial temperature gradient is associated to this diffusive mode. The convective heat transfer (stable regime) is characterized by the time constant $\tau_{\mathrm{dqm}} = L^2/v$ which is related to the fluid viscosity v and the characteristic length L. For water, as the working fluid, this time constant is about one second for a characteristic length (boundary layer thickness) of about 1 mm. The thermal diffusivity intervenes in the diffusive heat transfer and in this case the time constant τ_{dc} is about few seconds for the same characteristic length.

The change in the mode of heat transfer is all the more possible since both time constants (τ_{dqm} and τ_{dc}) are close, i.e. the Prandtl number of the fluid is not too far from unity. This means that both of these modes of heat transfer are in competition if thermal and hydrodynamical boundary layers have thicknesses of the same order. Thus, when we use a mixture of (water + glycol) as the working fluid [14], the Prandtl number $Pr = 30$, the convective mode is largely preponderant compared to the diffusive mode and in this case the probability of the occurrence of this instability is greatly reduced.

This phenomenological description could be completed by an analytical approach which uses the heat equation.

3.1.3. Analytical approach

The heat equation, in its local form, without any internal source term can be written as

$$\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} + v_r \frac{\partial T}{\partial r} + \frac{1}{r} v_\theta \frac{\partial T}{\partial \theta} = a \Delta T, \qquad (1)$$

where r, θ and z are, respectively, the radial, angular and axial coordinates, t is time, T the local temperature of a fluid element in the thermal boundary layer in the lower part of the cross section, a the thermal diffusivity of the fluid and Δ is the Laplacian operator. In this approach the variation in physical parameters versus temperature has been neglected.

In order to simplify the expression of Eq. (1), we write that

$$a \Delta T = q_{\mathrm{d}}, \qquad v_r \frac{\partial T}{\partial r} + \frac{1}{r} v_\theta \frac{\partial T}{\partial \theta} = q_{\mathrm{c}}, \qquad (2)$$

where q_{d} represents the heat supplied by diffusion to the fluid element and q_{c} the heat advected in a cross section, i.e. in r and θ directions; this corresponds to the convective transfer (free convection) in a cross section.

In the stable regime, experiments show that $\partial T/\partial t = 0$ and $\partial T/\partial z = 0$, so it can be deduced that: $q_{\mathrm{c}} = q_{\mathrm{d}}$. Locally, in the lower part of the cross section, the heat supplied by diffusion is transferred by convection in the cross section (free convection); the secondary flow (transverse velocities) advects energy from the bottom to the upper part of the cross section. The main flow, to which the v_z component is associated does not take part in heat transfer.

In the instabilities regime, in the lower part of the cross section, the experimental results show that (see phase 2 of Fig. 5) an increase of internal energy occurs which manifests itself through the linear increase in the temperature Tclb (with a positive slope about 1 °C/s). In this case $\partial T/\partial t > 0$ and $\partial T/\partial z > 0$, so the difference $[q_{\mathrm{d}} - q_{\mathrm{c}}]$ becomes positive, thus

$$\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} = q_{\mathrm{d}} - q_{\mathrm{c}} > 0.$$

As the result, heat supplied by diffusion is not entirely transferred by free convection in the cross section, the heat transfer mode becomes partially diffusive. This shows that the main flow becomes effective towards heat transfer through the term $v_z(\partial T/\partial z)$.

The energy acquired by the fluid during phase 2 (linear increase in Tclb) is abruptly dissipated during phase 3 (rapid decrease in temperature, slope about 10 °C/s); in this case $[q_{\mathrm{d}} - q_{\mathrm{c}}]$ is negative and so q_{c} is dominant again. Convective rolls undergo an extension in the cross section and their trajectory expands transiently, in the thermally stratified zone. This extension causes the upper part of the cross section to cool, in particular on the wall, by destruction of the thermal stratified zone.

Phases 2 and 3 are characterized by a difference between their time constants (in a ratio of about 5). This fact shows that the physical processes which govern these phases cannot be the same; indeed, the difference $[q_{\mathrm{d}} - q_{\mathrm{c}}]$, which is positive for phase 2, is governed by thermal diffusion while for phase 3, $[q_{\mathrm{d}} - q_{\mathrm{c}}]$ is negative and so heat transfer is convective. Transition from phase 2 to 3 corresponds to a second change of heat transfer mode, from diffusive to convective. This transition is induced by the reactivation of free convection due to the local heating of the fluid and so to the decrease of its volume mass in the bottom of the cross section.

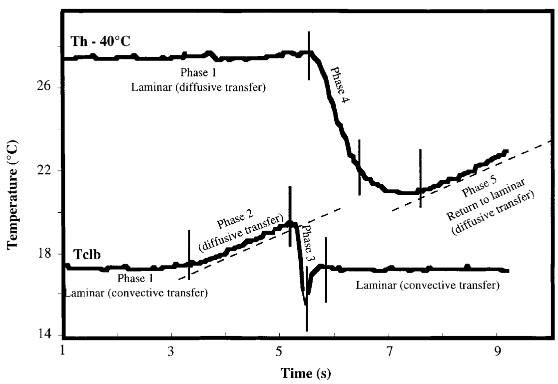

Fig. 5. Evolution of the wall temperature Th and the fluid temperature Tclb during the occurrence of the fluctuation ($Re = 1720$, $Ra = 303400$).

Phases 4 (decrease in temperature Th) and 3 which have time constant of the same order occur approximately simultaneously. This shows the coupling between phases 3 and 4. Phase 4 is followed by a return to the stable state (phase 5) with a time constant of about 10 s. So, it is the thermal diffusion which intervenes to reconstruct the thermal stratified zone, according to the initial state cohesion of events in the fluid, particularly in the thermal boundary layer.

The lower part is noteworthy; indeed:

- In this zone, the isochronous curves (Fig. 4) have a circular shape. This means that this instability is set off simultaneously for all the points located on a circular arc.
- The shape of the isochronous curves coincides with the shape of the isotherm curves.

These facts show that this instability is a global perturbation of the thermal boundary layer which does not modify the isotherm shapes. So, we can consider this situation as a perturbed state of the laminar regime.

We now perform a linear stability analysis of the basic state discussed above. Thus, we introduce $\mathrm{d}v_r$, $\mathrm{d}v_\theta$ and $\mathrm{d}v_z$ as perturbations of the velocity profile and ε as perturbation of the temperature, keeping in mind that they are dependent upon r, θ, z and time. The heat equation (1) linearized with respect to such perturbations may be written as

$$E + q'_\mathrm{d} = q'_\mathrm{c} - L \quad \text{with } E = \frac{\partial \varepsilon}{\partial t} + v_z \frac{\partial \varepsilon}{\partial z},$$
$$L = \mathrm{d}v_r \frac{\partial T}{\partial r} + \frac{1}{r}\mathrm{d}v_\theta \frac{\partial T}{\partial \theta}, \; q'_\mathrm{d} = a\Delta\varepsilon, \tag{3}$$
$$q'_\mathrm{c} = v_r \frac{\partial \varepsilon}{\partial r} + \frac{1}{r} v_\theta \frac{\partial \varepsilon}{\partial \theta},$$

where L and E correspond, respectively, to hydrodynamical and thermal fluctuations; the first line of (3) expresses the coupling of these perturbations. This coupling is effective only if the thicknesses of the hydrodynamical and thermal boundary layers are close enough; which is in agreement with the argument concerning the ratio of the momentum diffusivity to the thermal diffusivity, i.e. the Prandtl number which must be not too far from unity.

If we consider Eq. (3), in order for its terms to be of the same magnitude, the following conditions must be verified, that is:

$$\frac{\partial \varepsilon}{\partial r} \ll \frac{\partial T}{\partial r} \quad \text{and} \quad \frac{\partial \varepsilon}{\partial \theta} \ll \frac{\partial T}{\partial \theta}. \tag{4}$$

It is easy to see that these conditions are experimentally verified, however as $\partial T/\partial\theta$ is very small the ε perturbation must intervene simultaneously according to the isotherms curve in the lower part of the cross section.

4. Conclusion

In this paper, we have shown that thermal instability induced in the case of a mixed convection phenomenon is the result of a change in the heat transfer mode, in the lower part of the cross section, from a convective to a diffusive mode. This change occurs due to a competition between transverse (vertical) and axial (longitudinal) components of the velocity to advect heat supplied by the wall.

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